

# Remote Electronic Voting can be Efficient, Verifiable and Coercion-Resistant

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# Previous Work

(Juel, Catalano and Jakobsson, WPES 2005)

- JCJ formally defined the property of **coercion-resistance**, by considering possible attacks:
  - constrain a voter to cast given or random votes
  - force her to reveal her private data
  - vote on her behalf
  - force her to abstain
- Main idea: a coercer must be unable to distinguish a fake credential from a valid one.

⇒ for  $N$  ballots, the tallying complexity is in  $\mathcal{O}(N^2)$

# Motivations

	Linear complexity	Multiple elections	Practical for real polls	Completely anonymous
AFT07	✓	✗	✓	✓
AT13	✓	✓	✗	✓
CH11	✓	✓	✓	✗
SKHS11	✓	✓	✓	✗

# Building Blocks

- Designated Verifier Proof (DVP) which cannot be transferred:  
Only the designated verifier can be convinced by this proof
- Non-Interactive Zero-Knowledge Proof of Knowledge (NIZKP):  
Enable a prover to convince a verifier that he knows some secret
- ElGamal Cryptosystem
- Algebraic MAC Scheme
- Sequential Aggregate MAC Scheme

# ElGamal Cryptosystem

- Given  $\mathbb{G} = \langle g \rangle$  cyclic group of prime order  $p$ 
  - private key  $x$ , public key  $pk = g^x$
  - encryption of  $m$ :  $E_{pk}[m] = (g^r, mh^r)$
  - decryption of  $E_{pk}[m]$ :  $mh^r(g^r)^{-x}$
- Properties:
  - multiplicatively homomorphic:  $E_{pk}[m_1] \times E_{pk}[m_2] = E_{pk}[m_1 \times m_2]$
  - distribution of the private key (i.e. the decryption)
  - comparison of two ciphertexts via Plaintext Equivalence Test (PET):  
 $PET(E_{pk}[m_1], E_{pk}[m_2]) = 1$  if  $m_1 = m_2$  and 0 otherwise
  - easy re-encryption:  
 $E_{pk}[m] = (g^r, mh^r)$  can be transformed in  $E_{pk}[m]' = (g^{r+r'}, mh^{r+r'})$

# Algebraic MAC Scheme

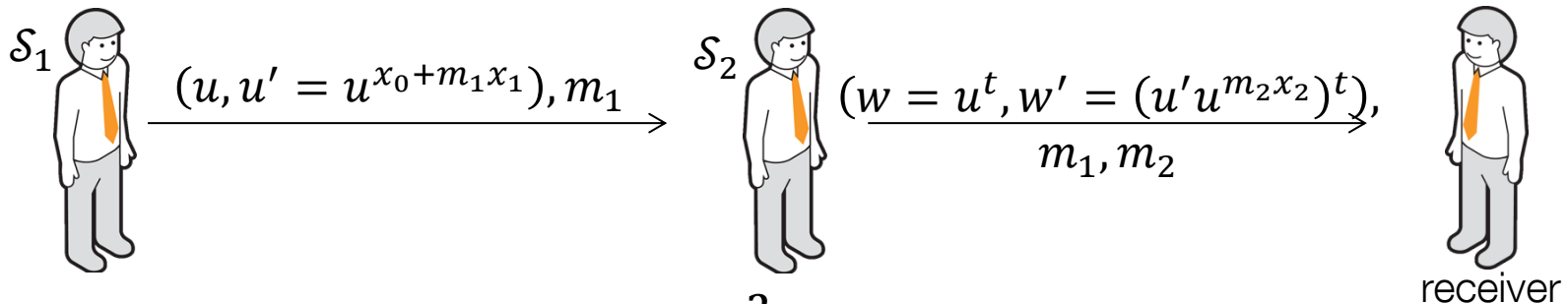
(Chase, Meiklejohn, Zaverucha, ACM CCS2014)

- $\text{Setup}(1^k)$ : Generate  $pp = (\mathbb{G}, p, g, h)$  such that
  - $\mathbb{G}$  cyclic group of prime order  $p$ , where DDH is hard
  - $g, h$  two of its generators
- $\text{KeyGen}(pp)$ :
  - secret key  $sk = (x_0, x_1, x_2)$
  - optionally, the public parameters ( $C_{x_0} = g^{x_0}h^x, X_1 = h^{x_1}, X_2 = h^{x_2}$ )
- $\text{MAC}(sk, m_1, m_2)$ :
  - choose  $u$  randomly
  - generate  $\sigma = (u, u')$  where  $u' = u^{x_0+m_1x_1+m_2x_2}$
- $\text{Verify}(sk, m_1, m_2, \sigma)$ :  $u \neq 1$  and  $u^{x_0+m_1x_1+m_2x_2} \stackrel{?}{=} u'$

Deciding whether  $(m, u, u' = u^{x_0+m_1x_1+m_2x_2})$  is a valid MAC on  $m$  is equivalent to the DDH problem.

# Our Sequential Aggregate MAC Scheme

- Setup:  $pp = (\mathbb{G}, p, g, h)$ 
  - $sk_1 = (x_0, x_1)$ , secret key of the first signer  $\mathcal{S}_1$
  - $sk_2 = x_2$ , secret key of the second signer  $\mathcal{S}_2$
  - $C_{x_0} = g^{x_0} h^x, X_1 = h^{x_1}, X_2 = h^{x_2}$ , associated public parameters
- Computation of MAC on  $m_1$  by  $\mathcal{S}_1$  and  $m_2$  by  $\mathcal{S}_2$ :



- Verification:  $w \neq 1$  and  $w' \stackrel{?}{=} w^{x_0+m_1x_1+m_2x_2}$

existentially unforgeable



# Our eVote Scheme

1. Setup
2. Registration
3. Voting
4. Tallying



Receive credential in order to cast a vote



Issue credentials in a distributed manner during the registration step



Force voters to make a particular vote and try to verify it



Jointly manage the tallying phase

# Security Model

- Registration occurs through an untappable channel  
⇒ no adversaries at this step
- Votes may be posted anonymously
- Bulletin Board is universally accessible
- Attacker may:
  - access to all public information
  - corrupt a subpart of the election authorities
  - coerce voters: requests secrets, forces a particular vote...

Voters trust their voting client.

# Set-Up

1. Setup
2. Registration
3. Voting
4. Tallying

- Set-Up:
  - $g, h, o$  generators of a cyclic group  $\mathbb{G}$  of prime order  $p$
  - registrars  $\mathcal{R}$ : share  $sk = (x_0, x_1)$ ,  $pk = (C_{x_0} = g^{x_0} h^x, X_1 = h^{x_1})$
  - talliers  $\mathcal{T}$ : share  $sk$  and an ElGamal keypair  $(T, \hat{T})$
- Registration:
  - credential  $(s, u, u')$ :
    - $s$  and  $u$  chosen randomly by  $\mathcal{R}$
    - $u' = u^{x_0 + sx_1}$  computed by  $\mathcal{R}$
  - in case of coercion, fake credential:  $(s', u, u')$  (DDH assumption)

# Registration

1. Setup
2. Registration
3. Voting
4. Tallying

- $\mathcal{R}$  jointly compute  $(u, u' = u^{x_0 + sx_1})$  with  $s, u$  cooperatively selected and prove its validity through a DVP:



$(s, u, u'), \text{DVP}$



- If a coercer asks to her credential, she can send a fake one:



$(s', u, u')$



The DVP can only convince the designated voter!

## More about our Ballot

- Credential:  $(s, u, u')$  where  $u' = u^{x_0 + sx_1}$
- Ballot:  $\langle E_T[v], w, w', E_T[w^s], o^s, P \rangle$ 
  - $(w, w')$  is a randomized credential s.t.  $w = u^l$  and  $w' = (u')^l$
  - $P$  is a pair of NIZKPs of validity:
    - $E_T[v]$  is an encryption of a valid vote
    - the voter knows:
      - the plaintext of  $E_T[w^s]$
      - the secret  $s$ , common both to  $E_T[w^s]$  and  $o^s$

# Voting (first election)

1. Setup
2. Registration
3. Voting
4. Tallying

- Vote under coercion:



$$\langle E_T[a], w, w', E_T[w^{S'}], o^{S'}, P \rangle$$

- Revote:



$$\langle E_T[b], w, w', E_T[w^S], o^S, P \rangle$$

Bulletin Board	
	⟨...⟩
	⟨ $E_T[a], w, w', E_T[w^{S'}], o^{S'}, P$ ⟩
	⟨...⟩
	⟨...⟩
	⟨...⟩
	⟨...⟩
	⟨ $E_T[b], w, w', E_T[w^S], o^S, P$ ⟩
	⟨...⟩
	⟨...⟩

# Tallying Phase [1/5]

1. Setup
2. Registration
3. Voting
4. Tallying

1. Discard ballots with invalid proofs

Bulletin Board (offline)
$\langle E_T[b], w_1, w_1', E_T[w_1^r], o^r, P \rangle$
$\langle E_T[b], w_2, w_2', E_T[w_2^s], o^s, P \rangle$
$\langle E_T[a], w_3, w_3', E_T[w_3^t], o^t, P \rangle$
$\langle E_T[b], w_4, w_4', E_T[w_4^{s'}], o^{s'}, P \rangle$
<del><math>\langle E_T[a], z_1, z_1', E_T[z_1^r], o^{r'}, P \rangle</math></del>
$\langle E_T[a], z_2, z_2', E_T[z_2^s], o^s, P \rangle$

# Tallying Phase [2/5]

1. Setup
2. Registration
3. Voting
4. Tallying

- 2. Remove duplicates votes  
⇒ ballots published using the same secret  $s$

Bulletin Board (offline)
$\langle E_T[b], w_1, w'_1, E_T[w_1^r], o^r \rangle$
<del><math>\langle E_T[b], w_2, w'_2, E_T[w_2^s], o^s \rangle</math></del>
$\langle E_T[a], w_3, w'_3, E_T[w_3^t], o^t \rangle$
$\langle E_T[b], w_4, w'_4, E_T[w_4^{s'}], o^{s'} \rangle$
$\langle E_T[a], z_2, z'_2, E_T[z_2^s], o^s \rangle$

Possible policy: keep the last one



# Tallying Phase [3/5]

1. Setup
2. Registration
3. Voting
4. Tallying

## 3. Reconstruction and checking of credentials

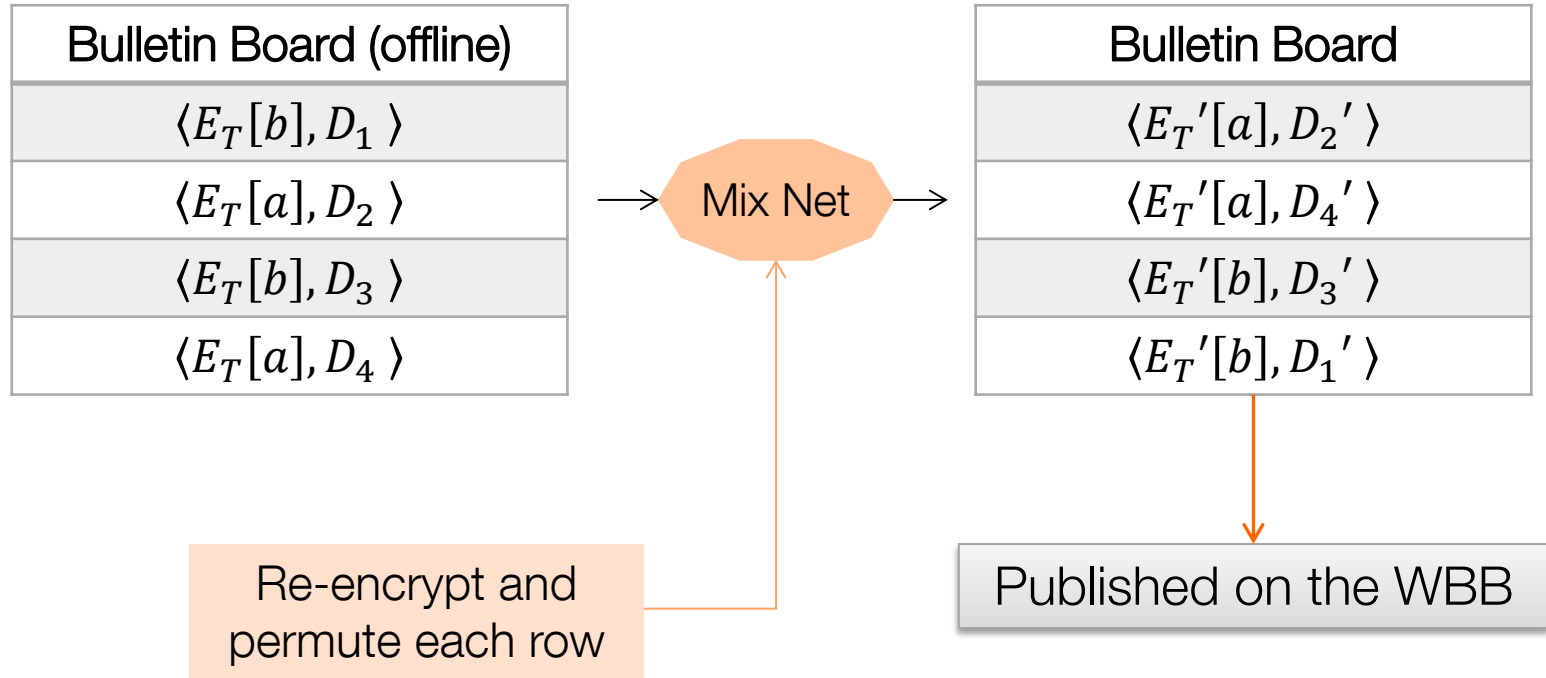
Bulletin Board (offline)
$\langle E_T[b], w_1, w'_1, E_T[w_1^r] \rangle$
$\langle E_T[a], w_3, w'_3, E_T[w_3^t] \rangle$
$\langle E_T[b], w_4, w'_4, E_T[w_4^{s'}] \rangle$
$\langle E_T[a], z_2, z_2', E_T[z_2^s] \rangle$

1. The authorities cooperatively compute  $E_T[w], E_T[w^{x_0}], E_T[w^s], E_T[(w^s)^{x_1}]$  in order to obtain:  
$$E_T[w^{x_0}] \times E_T[w^{sx_1}] = E_T[w^{x_0+sx_1}]$$
2. Then, power  $C = E_T[w^{x_0+sx_1}]/w'$  to a fresh random  $\alpha$  for the PET:  
$$D = C^\alpha$$
 should be equal to  $E_T[1]$

# Tallying Phase [4/5]

1. Setup
2. Registration
3. Voting
4. Tallying

## 4. Mix the ballots



# Tallying Phase [5/5]

1. Setup
2. Registration
3. Voting
4. Tallying

5. Identify valid votes by jointly decrypting  $D_i$ :
- If the plaintext is equal to 1, the ballot is valid and thus decrypted



# Multiple Elections and Credentials Revocation



- For a second election, registrars  $\mathcal{R}$ :
  - jointly generate an election identifier  $e_I$
  - compute a new pair of keys  $(x_2, X_2 = h^{x_2})$ , shared with the talliers  $\mathcal{T}$
  - publish an updated credential  $(w, w')$  for each eligible voter:  
 $(u, u' = u^{x_0 + sx_1})$  associated to the secret  $s$   
becomes  $(u^t, (u' u^{e_I x_2})^t) = (w, w' = w^{x_0 + sx_1 + e_I x_2})$

⇒ voting and tallying phases are unchanged

# Security

- A voter cannot prove her vote:
  - false and real credentials are indistinguishable
- No forced abstention:
  - votes cast using anonymous channel
- No forced randomization and impersonation:
  - voter can use fake credential for false vote and cast another one later
- Resistance to shoulder-surfing:
  - Re-vote policy: only the last might count

Our voting scheme satisfies:

- **eligibility** requirement through security properties of the MAC,
- **coercion-resistance** property under DDH assumption.

# Conclusion

- a Sequential Aggregate MAC Scheme **existentially unforgeable**
- Our new voting scheme for remote elections is:
  - publicly verifiable
  - efficient (linear time complexity)
  - coercion-resistant
  - allowing multiple elections and credentials revocation

Thank you

